

Nonclassical lattice solitons in optical lattice via Electromagnetically induced transparency

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An optical four-level atomic discrete system through optical induction is proposed. A theoretical scheme to produce nonclassical lattice solitons (NLS) in the system is presented with the use of the effects of enhanced self-phase modulation and the giant kerr effect in the electromagnetically induced transparency. The power density and the photon flux can be tuned to a very low level by the controlling field and the soliton can propagate with very slow group velocity. By changing the sign of the detuning Δ_1 , both in-phase and π out-of-phase NLSs can be produced in this system.

Optical fields propagating in couple waveguide arrays exhibit novel phenomena [1]. The light field shows great functionality in such an optical periodic system which are impossible appeared in the bulk. Recently, the behavior of nonclassical light field in couple waveguide arrays becomes an interesting issue [2, 3]. Nonclassical light is always a light field with single or few photons. In quantum information and quantum communication, photons are widely used as the information carrier to transfer signals. Generally, the number of photons are very few [4]. Therefore, such a optical periodic system provides an avenue to inventing new optical devices for quantum information processing. To avoid signals degradation during propagation, it is highly desirable to propagate the optical wave as a kind of soliton. Discrete or lattice soliton (LS) [5, 6] is a soliton mode in such a periodic waveguide arrays. It shows great potential in photonic network communications [7]. However, in the nonclassical light limit, the intensity of the soliton must be very low (i.e. at least few photons per soliton cross section per nanosecond). Traditional discrete systems such as kerr nonlinearity media [8], liquid crystals [9] photorefractive crystals [10] are very difficult to produce such kind of soliton because a required large nonlinear refractive index is always associated with intense light field. Therefore, it is important to realize a new optical periodic system that can produce a good controllability NLS in guiding these signals.

In this work, we will apply the technique of electromagnetically induce transparency (EIT) to overcome this difficulty in the atomic system. EIT media, which can be applied to inducing dissipation-free strong photon-photon interactions, can offer far larger third-order Kerr susceptibility and far smaller linear susceptibility than conventional nonlinear media [11]. Recently, ultraweak [12] and ultraslow [13] spatial and temporal solitons are realized by utilizing EIT in uniform atoms system, respectively. This opens the possibility of archiving NLS in atomic discrete systems.

In EIT medium, a dissipation-free optical discrete system can be generated by using optical induction technique. The energy diagram of the scheme is shown in FIG. 1 (a). It is similar to the traditional N-type system: the parties of $|1\rangle$, $|2\rangle$ are same and different from $|3\rangle$, $|4\rangle$. A weak probe wave E_P with Rabi-frequency $\Omega_P = \wp_{31}E_P/\hbar$ is acting on a resonant transition $|1\rangle \rightarrow |3\rangle$ with a single photon detuning Δ_1 . A running-wave field with a Rabi-frequency Ω_C is driving the atomic transition $|2\rangle \rightarrow |3\rangle$ with a detuning $\Delta_C = \Delta_1$, in other words, the two-photon detuning

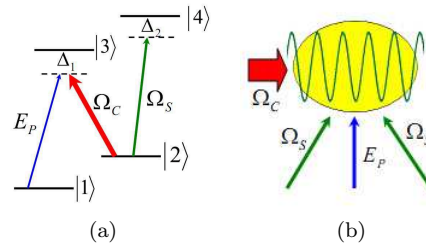


FIG. 1: (a) Energy diagram of the system. (b) Geometric configuration of the lights. Lattice-forming beams Ω_S and nonclassical light beam \hat{E}_P are co-propagating, The coupling light Ω_C is a p-wave, which is propagating transversely along the atoms

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$\delta = \Delta_1 - \Delta_C = 0$. An optical induction field with a Rabi-frequency Ω_S is inducing the transition $|2\rangle \rightarrow |4\rangle$ with a detuning Δ_2 . According to the previous studies [11], when the probe exactly at the two-photon resonance ($\delta = 0$) and the atoms are all in the ground state, the absorption to the probe is vanished and independent to the single-photon detuning. Meanwhile, the existence of the detuning Δ_1 and Δ_2 makes the probe experience an enhanced SPM produced by itself [14] and XPM by Ω_S [15], respectively.

We assume that $\Omega_P, \Omega_S \ll \Omega_C$, which results in the action of Ω_P and Ω_S can be treat as a perturbation. The interaction Hamiltonian between the light and atoms can be write as: $\hat{H} = \hat{H}_0 + \hat{H}_1$, where

$$\hat{H}_0 = \sum_{i=1}^5 \hbar \omega_i |i\rangle \langle i| - \frac{1}{2} [\Omega_C e^{-i\omega_C t} |3\rangle \langle 2| + H.c], \quad (1)$$

is the Hamiltonian operator without perturbation and

$$\hat{H}_1 = -\frac{1}{2} [\Omega_P e^{-i\omega_P t} |3\rangle \langle 1| + \Omega_S |4\rangle \langle 2| + H.c]. \quad (2)$$

is the perturbation's Hamiltonian. Therefore, the evolution equation of density matrix reads:

$$\dot{\rho}^{(n)} = -\frac{i}{\hbar} [\hat{H}_0, \rho^{(n)}] - \frac{i}{\hbar} [\hat{H}_1, \rho^{(n-1)}] - \frac{1}{2} [\Gamma, \rho^{(n)}]_+, \quad (3)$$

Where n designates the n -th step of the density matrix. The matrix elements $\langle i|\Gamma|j\rangle = \gamma_i \delta_{ij}$, where γ_i is the decay rate designating the population damping from the energy level $|i\rangle$. Assuming that all the atoms are populating at the ground state, in other words, $\rho_{11}^{(0)} = 1$ and $\rho_{ij}^{(0)} = 0$ (i or $j \neq 1$), and then the steady state solution of any step of the density matrix can be deduced by Eq. (3). Here we only deduce to the 3-step, the higher steps of the density matrix are ignored for their influences are tiny to the system. In the deduction, we assume that $\gamma_2 \approx \gamma_1 \approx 0$ and $\gamma_3 \approx \gamma_4 = \gamma$. Furthermore, to avoid the absorption from the atoms, we let $\Delta_1, \Delta_2 \gg \gamma, \Omega_C$. From Eq. (1)-(3) and the initial condition of $\rho^{(0)}$, one can obtain that:

$$\rho^{(1)} = \begin{pmatrix} 0 & \rho_{12}^{(1)} & 0 & 0 \\ \rho_{21}^{(1)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (4)$$

and:

$$\rho^{(2)} = \begin{pmatrix} 0 & 0 & 0 & \rho_{14}^{(2)} \\ 0 & 0 & \rho_{23}^{(2)} & 0 \\ 0 & \rho_{32}^{(2)} & 0 & 0 \\ \rho_{41}^{(2)} & 0 & 0 & 0 \end{pmatrix} \quad (5)$$

where $\rho_{21}^{(1)} = \rho_{12}^{(1)*} \approx -\Omega_P \Omega_C^* / |\Omega_C|^2$, $\rho_{32}^{(2)} = \rho_{23}^{(2)*} \approx -|\Omega_P|^2 \Omega_C / 2|\Omega_C|^2 \Delta_1$, and $\rho_{41}^{(2)} = \rho_{14}^{(2)*} \approx -\Omega_C^* \Omega_P \Omega_S / 2|\Omega_C|^2 \Delta_2$. And then obtain that:

$$\rho_{31}^{(3)} \approx \frac{|\Omega_S|^2}{2\Delta_2 |\Omega_C|^2} \Omega_P - \frac{|\Omega_P|^2}{2\Delta_1 |\Omega_C|^2} \Omega_P, \quad (6)$$

$$\rho_{42}^{(3)} = \frac{|\Omega_P|^2}{4\tilde{\omega}_{42}\tilde{\omega}_{43} - |\Omega_C|^2} \left[\frac{1}{\Delta_1} - \frac{1}{\Delta_2} \right] \Omega_S, \quad (7)$$

where $\tilde{\omega}_{42} = \Delta_2 - i\gamma_{42}$ and $\tilde{\omega}_{43} = \Delta_2 - \Delta_1 - i\gamma_{43}$. The paraxial steady-state propagation equations of E_P and E_S in the slowly varying envelopes read as:

$$2ik_P \frac{\partial}{\partial z} E_P = -\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) E_P - \frac{k_P^2}{\epsilon_0} 2N \wp_{31} \rho_{31}^{(3)}, \quad (8)$$

$$2ik_S \frac{\partial}{\partial z} E_S = -\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) E_S - \frac{k_S^2}{\epsilon_0} 2N \wp_{42} \rho_{42}^{(3)}, \quad (9)$$

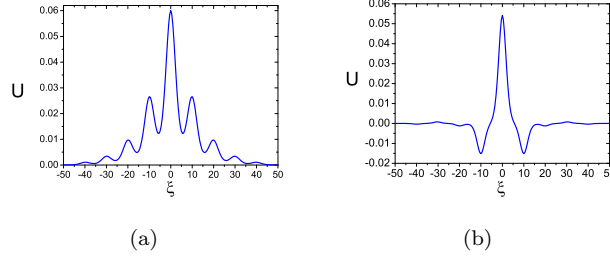


FIG. 2: (a) In-phase LS with $\kappa = -2.5$ ($\Delta_1 = -100\gamma$) and $V_0 = 0.5$ and $P = \int |U|^2 d\xi = 0.023$. (b) π out-of-phase LS with $\kappa = 2.5$ ($\Delta_1 = 100\gamma$), $V_0 = 0.75$ and $P = 0.0106$.

where N is the density of the atoms. According to Eq. (7), if we let $|\Delta_1| = |\Delta_2|$, the propagation dynamics of E_S can be treated as linear by the condition of $\Delta_1, \Delta_2 \gg \gamma_i, \Omega_C$. The reason is: (1) when $\Delta_1 = \Delta_2$, it is obvious obtained that $\rho_{42}^{(3)} = 0$; (2) when $\Delta_1 = -\Delta_2$, $\rho_{42}^{(3)}/\rho_{31}^{(3)} \sim |\Omega_C|^2/\Delta_1^2 \ll 1$, which results in that the refractive index modulation from E_P can be neglected. Under these circumstances, E_P propagates nonlinearly with the modulation from E_S , while E_S propagates nearly linearly. This relationship is the key element to form a discrete system via optical induction [5]. Therefore, we can apply $|E_S|^2$ to form the pattern of the lattice waveguide arrays (See in FIG. 1 (b)). Substituting $\rho_{31}^{(3)}$ in Eq. (8), we have:

$$i \frac{\partial}{\partial \zeta} U = -\frac{1}{2} \nabla_{\perp}^2 U + V(\xi, \eta) U + \kappa |U|^2 U, \quad (10)$$

where $\nabla_{\perp}^2 = (\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2})$ with $\zeta = zk_0$, $\xi = xk_0$, $\eta = yk_0$; $U = \Omega_P/\Omega_C$ is the dimensionless field, and,

$$V(\xi, \eta) = -\frac{N|\wp_{31}|^2}{2\epsilon_0 \hbar \Delta_2} \frac{|\Omega_S(\xi, \eta)|^2}{|\Omega_C|^2}, \kappa = \frac{N|\wp_{31}|^2}{2\epsilon_0 \hbar \Delta_1}. \quad (11)$$

And then, E_P experiences two kinds of refractive index modifications: (1) a periodical index change induced by Ω_S (via XPM); (2) an index change induced by E_P itself (via SPM). Here, Ω_S could be viewed as a linear potential added on E_P . The sign of detuning Δ_1 defines that E_P experiences self-focusing ($\Delta_1 < 0$) or self-defocusing ($\Delta_1 > 0$); while the sign of detuning Δ_2 defines that the energy of the soliton field is located on a lattice waveguide site (when $\Delta_2 > 0$) or between lattice sites (when $\Delta_2 < 0$).

In the 1D case, choose the lattice-forming wave to be $|\Omega_S|^2 = |\Omega_S^0|^2 \sin^2(\pi\xi/d)$, and the periodical potential in Eq. (11) can be written as $V = V_0^2 \sin^2(\pi\xi/d)$, where $V_0^2 = N|\wp_{31}|^2|\Omega_S^0|^2/2\epsilon_0 \hbar \Delta_2 |\Omega_C|^2$. Assuming the probe experiences self-focusing ($\Delta_1 < 0$), the bound state solution of Eq. (10) can be obtained by the imaginary-time propagation (ITP) methods. The parameters are chosen referring to the Rb atoms as: $\wp_{31} = 3.0 \times 10^{-29}$ D, $\gamma \approx 6$ MHz and $\lambda_P = 795$ nm. For convenience, other parameters are chosen as: $d = 10$, $|\Omega_S^0|^2/|\Omega_C|^2 = 0.1$ with $\Omega_C = 2\gamma$, $\Delta_1 = \Delta_2 = -100\gamma$ and $N = 3 \times 10^{15} \text{ cm}^{-3}$, which are tunable in practice. Then the normalized parameters σ and V_0 are obtained: $\sigma = 2.5$ and $V_0 = 0.5$. In the simulations, we chose the normalized transverse power of the field to be $P = \int |U|^2 d\xi = 0.023$. The numerical solutions are plotted in FIG. 2(a).

The power density of the soliton can be estimated by $I_P = \frac{1}{2} \epsilon_0 c \frac{\hbar^2 \Omega_C^2}{L_P |\wp_{31}|^2} P$, where L_P is the transverse beam-width of the probe. Then the photon flux which passes through the soliton can be estimated by $I_P/\hbar\omega_P$ [12]. Moreover, the group velocity of the probe can also be estimated by [16]: $V_g \approx \frac{\hbar c \epsilon_0}{2\omega_P} \frac{|\Omega_C|^2}{|\wp_{31}|^2 N}$. They are all in proportion to $|\Omega_C|^2$. If we chose $\Omega_C = 2\gamma$, one can obtain that $I_P = 1.81 \times 10^{-4} \text{ mW/cm}^2$, photon flux is: $7 \text{ mm}^{-2} \text{ ns}^{-1}$, and group velocity is: 3.15 mm/s , which have reached the nonclassical light region and the soliton can be termed as a NLS.

If the detuning $\Delta_1 > 0$, E_P will act as self-defocusing in the medium. With the periodic modulation induced by Ω_S , Eq. (10) has π out-of-phase bright NLS (at the edge of the Brillouin zone) solutions. Such NLSs can be obtained by angle incident with the Bloch momentum lies in the vicinity of the edge of the first Brillouin zone. Diffraction in this condition is negative and self-defocusing nonlinearity is needed in trapping the fields [6, 10]. FIG. 2(b) plots a profile of a π out-of-phase DS with $\kappa = 2.5$ ($\Delta_1 = 100\gamma$), $V_0 = 0.75$ and $P = 0.0106$.

In conclusion, we demonstrate that the optical induction technique can be applied to build an optical atomic discrete system in an atomic system through the technique of EIT. The probe field is experiencing the Kerr nonlinear self modulation and periodical index modulation of the lattice-forming wave, respectively. By changing the detuning Δ_1 , both in-phase and π out-of-phase NLSs can be produced in this system. The technique illustrated in this paper

involves manipulating resonant optical properties of an atomic medium, and allow people to coherently control the propagation of a nonclassical light beam. By using this technique, many complex 2D LS including quasi-crystals LS [17], defect mediated LS [18], honeycomb LS [19], vortex [20] or vortex-ring [21] LS via optical induction can all be realized in such system.

Yongyao Li thanks for Prof. Jianying Zhou for useful discussion. This work is supported by the NSFC (10934011, 10774193, 10947126, ZA091201).

Appendix A

Here, we provide the detail deduction of the results in Eq. (4)-(7). According to the Eq. (3) and the initial condition:

$$\rho^{(0)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (\text{A1})$$

the slowly varying amplitude evolution equations of the one-step density matrix elements are read as:

$$\dot{\rho}_{41}^{(1)} = -i\tilde{\omega}_{41}\rho_{41}^{(1)}, \quad (\text{A2})$$

$$\dot{\rho}_{42}^{(1)} = -i\tilde{\omega}_{42}\rho_{42}^{(1)} - \frac{i}{2}\Omega_C\rho_{43}^{(1)}, \quad (\text{A3})$$

$$\dot{\rho}_{43}^{(1)} = -i\tilde{\omega}_{43}\rho_{43}^{(1)} - \frac{i}{2}\Omega_C^*\rho_{42}^{(1)}, \quad (\text{A4})$$

$$\dot{\rho}_{44}^{(1)} = -\gamma_4\rho_{44}^{(1)}, \quad (\text{A5})$$

$$\dot{\rho}_{31}^{(1)} = -i\tilde{\omega}_{31}\rho_{31}^{(1)} + \frac{i}{2}\Omega_C\rho_{21}^{(1)} + \frac{i}{2}\Omega_P\rho_{11}^{(0)}, \quad (\text{A6})$$

$$\dot{\rho}_{32}^{(1)} = -i\tilde{\omega}_{32}\rho_{32}^{(1)} - \frac{i}{2}\Omega_C(\rho_{33}^{(1)} - \rho_{22}^{(1)}), \quad (\text{A7})$$

$$\dot{\rho}_{33}^{(1)} = -\gamma_3\rho_{33}^{(1)} + \frac{i}{2}(\Omega_C\rho_{23}^{(1)} - \Omega_C^*\rho_{32}^{(1)}), \quad (\text{A8})$$

$$\dot{\rho}_{21}^{(1)} = -i\tilde{\omega}_{21}\rho_{21}^{(1)} + \frac{i}{2}\Omega_C^*\rho_{31}^{(1)}, \quad (\text{A9})$$

$$\dot{\rho}_{22}^{(1)} = -\gamma_2\rho_{22}^{(1)} + \frac{i}{2}(\Omega_C^*\rho_{32}^{(1)} - \Omega_C\rho_{23}^{(1)}), \quad (\text{A10})$$

$$\dot{\rho}_{11}^{(1)} = -\gamma_1\rho_{11}^{(1)}, \quad (\text{A11})$$

$$\rho_{ij}^{(1)} = \rho_{ji}^{*(1)}, \quad (\text{A12})$$

where: $\tilde{\omega}_{41} = \delta + \Delta_2 - i\gamma_{41}$, $\tilde{\omega}_{31} = \Delta_1 - i\gamma_{31}$, $\tilde{\omega}_{32} = \Delta_C - i\gamma_{32}$, $\tilde{\omega}_{21} = \delta - i\gamma_{21}$, and $\gamma_{ij} = (\gamma_i + \gamma_j)/2$. Here, we assume $\gamma_1 \approx \gamma_2 \approx 0$ and $\gamma_3 \approx \gamma_4 = \gamma$. Because of $\tilde{\omega}_{21} \approx 0$, from Eq. (A6) and (A9), the steady state solution of $\rho_{31}^{(1)}$ and $\rho_{21}^{(1)}$ are:

$$\begin{aligned} \rho_{31}^{(1)} &= \frac{2\tilde{\omega}_{21}\Omega_P}{4\tilde{\omega}_{31}\tilde{\omega}_{21} - |\Omega_C|^2} = \rho_{13}^{*(1)} \approx 0 \\ \rho_{21}^{(1)} &= \frac{\Omega_C^*\Omega_P}{4\tilde{\omega}_{31}\tilde{\omega}_{21} - |\Omega_C|^2} = \rho_{12}^{*(1)} \approx -\frac{\Omega_C^*\Omega_P}{|\Omega_C|^2}. \end{aligned} \quad (\text{A13})$$

And from this set of equations, one can easily obtain other steady state solution of one-step density matrix elements are all equals 0. Then the results in Eq. (4) can be obtained.

The slowly varying amplitude evolution equations of the 2-step density matrix elements are as below:

$$\dot{\rho}_{41}^{(2)} = -i\tilde{\omega}_{41}\rho_{41}^{(2)} + \frac{i}{2}\Omega_S\rho_{21}^{(1)}, \quad (\text{A14})$$

$$\dot{\rho}_{42}^{(2)} = -i\tilde{\omega}_{42}\rho_{42}^{(2)} - \frac{i}{2}\Omega_C\rho_{43}^{(2)}, \quad (\text{A15})$$

$$\dot{\rho}_{43}^{(2)} = -i\tilde{\omega}_{43}\rho_{43}^{(2)} - \frac{i}{2}\Omega_C^*\rho_{42}^{(2)}, \quad (\text{A16})$$

$$\dot{\rho}_{44}^{(2)} = -\gamma_4\rho_{44}^{(2)}, \quad (\text{A17})$$

$$\dot{\rho}_{31}^{(2)} = -i\tilde{\omega}_{31}\rho_{31}^{(2)} + \frac{i}{2}\Omega_C\rho_{21}^{(2)}, \quad (\text{A18})$$

$$\dot{\rho}_{32}^{(2)} = -i\tilde{\omega}_{32}\rho_{32}^{(2)} - \frac{i}{2}\Omega_C(\rho_{33}^{(2)} - \rho_{22}^{(2)}) + \frac{i}{2}\rho_{12}^{(1)}, \quad (\text{A19})$$

$$\dot{\rho}_{33}^{(2)} = -\gamma_3\rho_{33}^{(2)} + \frac{i}{2}(\Omega_C\rho_{23}^{(2)} - \Omega_C^*\rho_{32}^{(2)}), \quad (\text{A20})$$

$$\dot{\rho}_{21}^{(2)} = -i\tilde{\omega}_{21}\rho_{21}^{(2)} + \frac{i}{2}\Omega_C^*\rho_{31}^{(2)}, \quad (\text{A21})$$

$$\dot{\rho}_{22}^{(1)} = -\gamma_2\rho_{22}^{(2)} + \frac{i}{2}(\Omega_C^*\rho_{32}^{(2)} - \Omega_C\rho_{23}^{(2)}), \quad (\text{A22})$$

$$\dot{\rho}_{11}^{(1)} = -\gamma_2\rho_{11}^{(2)}, \quad (\text{A23})$$

$$\rho_{ij}^{(2)} = \rho_{ji}^{*(2)}. \quad (\text{A24})$$

From Eq. (A20) and Eq. (A22), the level $|3\rangle$ and $|2\rangle$ with Ω_C can be treated as a two-level system. Then we have:

$$\dot{W}_{32}^{(2)} = \dot{\rho}_{33}^{(2)} - \dot{\rho}_{22}^{(2)} = -\Gamma_{32}(W_{32}^{(2)} - W_{32}^{(0)}) + i(\Omega_C\rho_{23}^{(2)} - \Omega_C^*\rho_{32}^{(2)}), \quad (\text{A25})$$

where $\Gamma_{32} \approx \gamma_3$ is the transverse relaxation rate between $|2\rangle$ and $|3\rangle$ and $W_{32}^{(0)} = \rho_{33}^{(0)} - \rho_{22}^{(0)} = 0$. Therefore, the steady state solution of $W_{32}^{(2)}$ can be obtained by:

$$W_{32}^{(2)} = \rho_{33}^{(2)} - \rho_{22}^{(2)} = \frac{i}{\Gamma_{32}}(\Omega_C\rho_{23}^{(2)} - \Omega_C^*\rho_{32}^{(2)}). \quad (\text{A26})$$

Substituted Eq. (A26) into Eq. (A19) and its conjugate, under the steady state condition, one can obtain an equations set about $\rho_{32}^{(2)}$ and $\rho_{23}^{(2)}$ as below:

$$\begin{cases} \tilde{\omega}_{32}\rho_{32}^{(2)} - \frac{1}{2}\Omega_C W_{32}^{(2)} = \frac{1}{2}\Omega_P\rho_{12}^{(1)} \\ \tilde{\omega}_{32}^*\rho_{23}^{(2)} - \frac{1}{2}\Omega_C^* W_{32}^{(2)} = \frac{1}{2}\Omega_P^*\rho_{21}^{(1)} \end{cases} \quad (\text{A27})$$

The steady state solution of $\rho_{32}^{(2)}$ can be obtained from this equations set:

$$\rho_{32}^{(2)} = \rho_{23}^{*(2)} = -\frac{|\Omega_P|^2\Omega_C\tilde{\omega}_{32}}{2(|\tilde{\omega}_{32}|^2 + |\Omega_C|^2)|\Omega_C|^2} \quad (\text{A28})$$

From Eq. (A14) the steady state solution of $\rho_{41}^{(2)}$ can be obtained by:

$$\rho_{41}^{(2)} = \rho_{14}^{*(2)} = -\frac{\Omega_C^* \Omega_P \Omega_S}{2\tilde{\omega}_{41} |\Omega_C|^2}. \quad (\text{A29})$$

Under the condition of $\Delta_1, \Delta_2 \gg \gamma_{32}, |\Omega_C|^2$, $\rho_{32}^{(2)}$ and $\rho_{41}^{(2)}$ can be simplified to:

$$\rho_{32}^{(2)} \approx -\frac{|\Omega_P|^2 \Omega_C}{2|\Omega_C|^2 \Delta_1}; \rho_{41}^{(2)} \approx -\frac{\Omega_C^* \Omega_P \Omega_S}{2|\Omega_C|^2 \Delta_2}. \quad (\text{A30})$$

Other steady solutions of 2-step density matrix element are all equal to 0. Then the result of Eq. (5) can be obtained.

Because we only need the steady solution of $\rho_{31}^{(3)}$ and $\rho_{42}^{(3)}$ in the 3-step density matrix elements. Therefore in the 3-step equations, we only need 2 equations sets:

$$\begin{cases} \dot{\rho}_{42}^{(3)} = -i\tilde{\omega}_{42}\rho_{42}^{(3)} - \frac{i}{2}\Omega_C\rho_{43}^{(3)} \\ \dot{\rho}_{42}^{(3)} = -i\tilde{\omega}_{43}\rho_{43}^{(3)} - \frac{i}{2}(\Omega_C^*\rho_{42}^{(3)} + \Omega_P^*\rho_{41}^{(2)} - \Omega_S\rho_{23}^{(2)}); \end{cases} \quad (\text{A31})$$

and

$$\begin{cases} \dot{\rho}_{31}^{(3)} = -i\tilde{\omega}_{31}\rho_{31}^{(3)} - \frac{i}{2}\Omega_C\rho_{21}^{(3)} \\ \dot{\rho}_{21}^{(3)} = -i\tilde{\omega}_{21}\rho_{21}^{(3)} + \frac{i}{2}(\Omega_C^*\rho_{31}^{(3)} - \Omega_P\rho_{23}^{(2)} + \Omega_S^*\rho_{41}^{(2)}). \end{cases} \quad (\text{A32})$$

By solving these two equations sets, one can obtain the results in Eq. (6) and Eq. (7).

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